ADVANCED UNDERGRADUATE LABORATORY

QE

Quantum Entanglement and Bell's Inequality

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Contents

1 Introduction to Bell's Experiment

In this lab you will be performing a well-known experiment that demonstrates a fundamental aspect of quantum mechanics, namely non-locality. In this context, non-locality can be loosely defined as correlations between spatially separated events. The basic quantum-mechanical objects that will be studied are photons, produced by a strong laser. By sending the beam of photons through a crystal with special optical properties, two weak beams are generated and exit the crystal an angle of 3° (see Fig. 1). Due to the process inside the crystal, the photons in the two weak beams are correlated in a quantum mechanical sense, which is referred to as **entanglement**. It is the goal of this lab to demonstrate non-locality by measuring the properties of these two entangled beams using using polarizers and single photon detectors. In particular, a famous result known as **Bell's inequality** puts a constraint on how correlated the two photons can be under the assumption that nature is local [1]. However, quantum mechanics predicts that this inequality is broken. In this experiment you will test this prediction. If the inequality is broken, then you will have shown that nature is truly non-local.

2 Theory

2.1 Polarizers and Bell's Inequality

In this manual, a minimal introduction to the necessary background for the experiment will be given. To gain more insight into the physics underlying this experiment, the reader is encouraged to refer to Dehlinger [1] for a clear and concise review of all the relevant aspects of the experiment

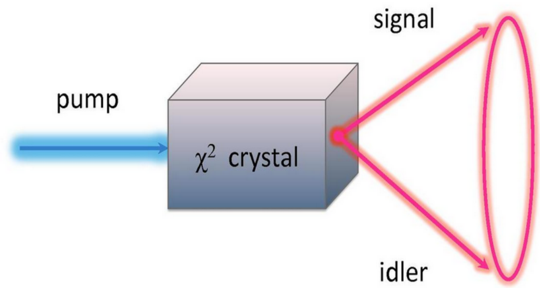

Figure 1: A illustration of the process which generates the two weak entangled beams of light from strong laser light.

you will be performing. This paper will be accessible to students who have taken basic quantum mechanics (you should be comfortable with bra-ket notation).

We now move on to discuss the basic theory behind the experiment. To measure properties of the entangled photons, each entangled beam will be sent through a polarizer before being measured by a single photon detector (see Fig. 3). Every time a photon reaches the polarizer, it passes through with a certain probability that depends on the orientation of the **transmission axis**, which can be changed by rotating the polarizer. For example, if you send an unentangled and vertically polarized photon into a polarizer with a vertical transmission axis, it will always pass through. If you then rotate the transmission axis, the probability of the photon passing through will drop, reaching zero when the transmission axis is horizontal.

Now consider a pair of photons, one in each of the two entangled beams. When the two photons reach their respective polarizers, there are four possible outcomes: they are both transmitted, they are both absorbed, or one of them is absorbed and the other is not (and vice versa). Transmission corresponds to measuring the polarization to be vertical in the direction of the transmission axis, and absorption corresponds to a measurement outcome of horizontal polarization. With this in mind, we denote transmission and absorption as V and H, and the probabilities for these four events can be written as $P_{VV}(a,b)$, $P_{HH}(a,b)$, $P_{VH}(a,b)$, and $P_{HV}(a,b)$. Here a and b are the angles of transmission axes of the two polarizers with respect to the vertical. For further clarification of this notation, refer to Dehlinger [1].

Bell's inequality is expressed in terms of these probabilities. First, we define a measure of the correlation between the two entangled beams,

$$E(a,b) \equiv P_{VV}(a,b) + P_{HH}(a,b) - P_{VH}(a,b) - P_{HV}(a,b). \tag{1}$$

As mentioned in Dehlinger [1], this measure of correlation varies between $+1$ when the polarizations of the two photons always agree, and -1 when the are always measured to be different. From $E(a,b)$, we can now define a quantity S:

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b'). \tag{2}$$

Bell's inequality then says that for any theory of nature that is entirely local, we have

$$|S| \leq 2. \tag{3}$$

The proof of this fundamental result can be found in the appendix of [1]. As the author points out, S does not have a clear physical meaning. What is important, however, is that we now have a concrete way of testing whether nature is non-local. It turns out that quantum mechanics predicts a maximum value of $S = 2\sqrt{2}$ for a particular entangled state known as the **Bell State**, and a particular choice of polarizer angles. If we measure S for this state, we can thus determine if the prediction is wrong (and thus quantum mechanics is incorrect), or quantum mechanics is correct and nature is truly non-local.

2.2 The Bell State

The general state of the entangled light generated by the crystal is

$$|\psi\rangle = \cos\theta\, |V\rangle_1 |V\rangle_2 + e^{i\phi} \sin\theta\, |H\rangle_1 |H\rangle_2\,, \tag{4}$$

where $|V\rangle$ and $|H\rangle$ refer to vertical and horizontal polarization states. In this state, θ dictates the relative amount of the HH vs VV states in the entangled light, and ϕ is the relative phase between the HH and VV components. To change the entangled state coming *out* of the crystal, you must change the state of the laser light going *in to* the crystal. This can be done using two components, called the Quarter and Half Wave Plates (QWP and HWP), as explained in section 3 below. The entangled Bell state we are interested in corresponds to $\theta = 45°$ and $\phi = 0$:

$$|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_1 |V\rangle_2 + |H\rangle_1 |H\rangle_2). \tag{5}$$

3 Apparatus

- **Pump laser** This is the laser that produces the photons for the experiment. Note that the power of this laser is much greater than the power of the entangled beams.

- **Laser Controller** This box allows the user to turn on/off the laser beam, and adjust its intensity by changing the current flowing into the laser. When the laser is on, the current should be set to 80mA for this demo. To operate, first push on the POWER button. To turn on the laser, then push on the OUTPUT button below the large knob. Finally, turn up large knob to set the current.

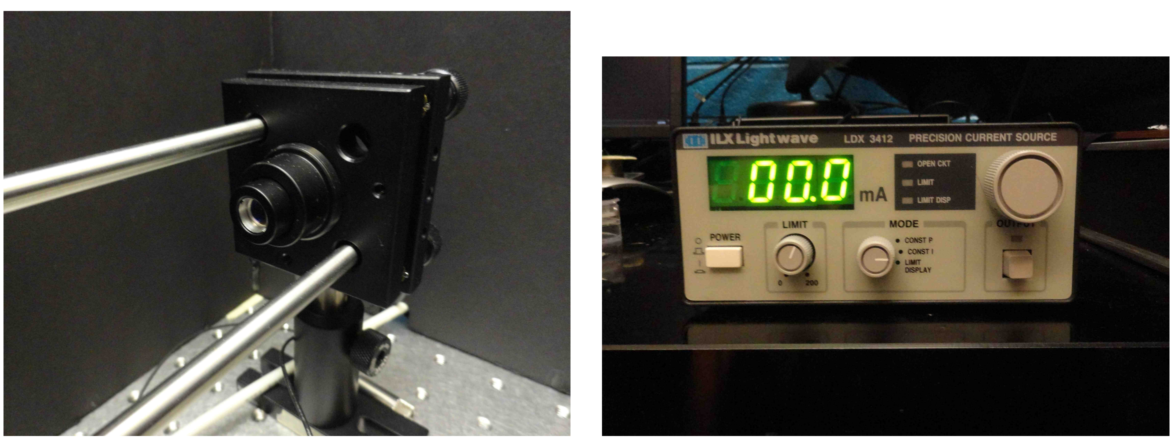

Figure 2: Left: the pump laser. Right: laser controller

- **Quarter wave plate (QWP) and Half wave plate (HWP)** Both plates are inside the safety enclosure. The laser light first passes through the QWP, then the HWP, then the crystal, before exiting the safety enclosure. By rotating the wave plates you can change the properties of the laser light. In this way, these components allow you to change the 'state' of the two entangled beams generated by the crystal.

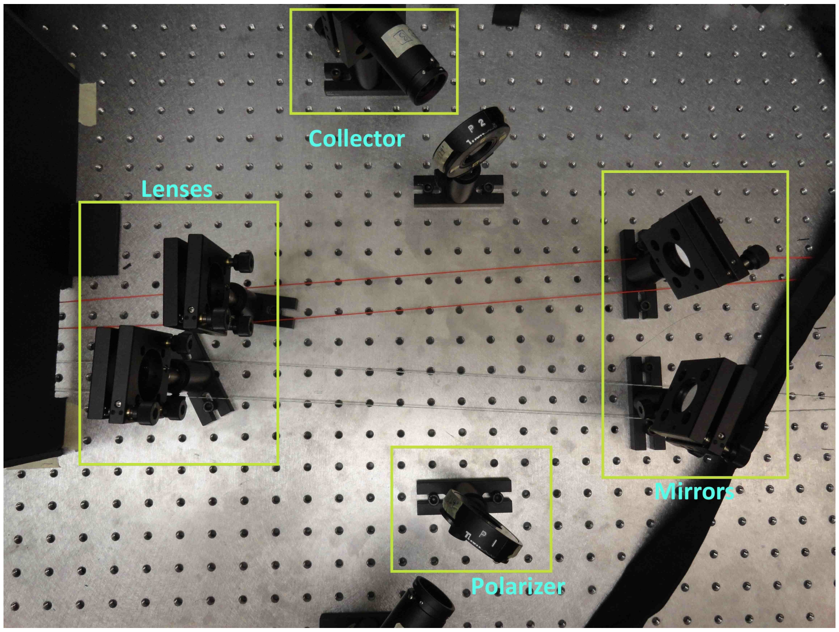

Figure 3: The collectors, mirrors, and polarizers

- **Lenses** The purpose of the two lenses is to focus the entangled photon beams before entering the collectors. Each lens is mounted in an optical cage, and has a focal length of 25 cm. Avoid touching these lenses throughout the experiment.
- **Linear polarizers** To measure properties of the entangled beams, and in particular the 'correlation' between them, the entangled beams are sent through linear polarizers.
- **Collectors** The two collectors are used to collect the incoming entangled photon beams. Each collector has a small lens near the back that focuses the beams onto an optical fibre, which leads to the single photon detectors. On the back of each collector there are three knobs for adjusting the direction that the collector points. These knobs, together with the adjustment knobs on the mirrors, are the tuneable degrees of freedom used for alignment of the setup. Also note that there are irises on the collectors to block out light from reaching the sensitive detectors.
- **Mirrors** Together with the adjustable orientation of each collector, the two mirrors are used for alignment of the entangled beams.
- **LED Lamp** This lamp can be turned on and off without affecting the apparatus (which is generally sensitive to light). This lamp is found beside the computer.

4 LabVIEW Program

In this section, we outline the major components of the LabVIEW interface that you will be interacting with throughout the experiment. Fig. 4 shows a screenshot of the interface. This program can be accessed as **detection.vi** on the computer beside the experimental setup. NOTE: the LabVIEW software reports counts for several detectors that are not in use. The student should only use the two channels that are in use for the experiment.

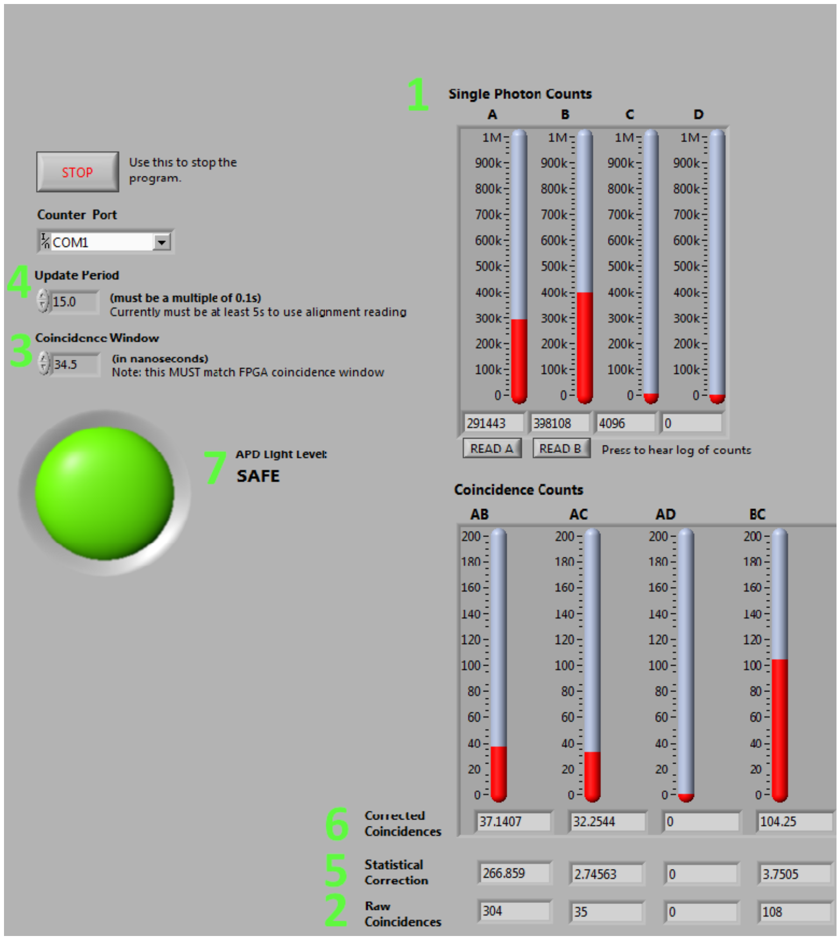

Figure 4: The LabVIEW screen

1. **Single photon counts** This indicates the total number of single photons detected by a particular detector within the given time window. This number does not include any post-processing, but simply indicates the raw number of photons counted by that detector.

2. **Raw coincidences** This indicates the number of times that two photons each hit the two detectors within a pre-selected time window (coincidence window). For example, if were looking at the coincidence counts AB for detectors A and B, and we have a coincidence window of τ seconds, the coincidence counts are the number of times a photon hits detector A and another photon hits detector B, and both hits occur within τ seconds of each other.

3. **Coincidence window** Time window that dictates the tolerance for what defines a coincidence count. See description for coincidence counts above.

4. **Update period** Time window that sets how often the photon counts are refreshed on the LabVIEW interface.

5. **Statistical correction** The number of coincidence counts indicated by the software includes the effects of ambient (background) photons that we are not studying, that may coincidentally hit the two detectors within the coincidence window. This would cause an artificially high number of coincidence counts. Using the statistical estimates of coincidence counts due to ambient photons, the software attempts to correct for these excess background counts. The statistical correction indicates the amount by which the coincidence counts need to be corrected to reduce the effect of ambient photons.

6. **Corrected coincidences** Number of coincidence counts due to entangled photons detected, after applying the statistical correction to remove coincidence counts due to background effects. When the laser is off and the detectors are only measuring classical ambient light, the corrected counts should be zero, because all the coincidence counts are statistical (i.e. random).

7. **Large green/red indicator** This large circle turns red if photon measurements on the LabVIEW program have reached their maximum displayable values (i.e. they max out). Note that this is a software warning, and does not *necessarily* mean the detectors are being damaged. Nonetheless, you should pay attention to the indicator and try not to flood the detectors.

5 Overview of Experiment

The end goal of this experiment is to calculate S, and compare with Bell's inequality, $|S| \leq 2$. This will require three main tasks:

1. **Aligning the setup:** you must first align the setup without polarizers, so that the two entangled beams are detected. The goal here is to obtain a suitably high rate of single photon counts and coincidence counts.

2. **Tune the Bell state:** the next step is to use the quarter and half-wave plates to adjust the properties of the entangled light so that it comes close to the Bell state. Recall that this is done because quantum theory predicts $S = 2\sqrt{2} > 2$ for the Bell state, violating Bell's inequality. While the Bell state produces the maximum S value, Bell's inequality can still be broken for a state that is not exactly the Bell state.

3. **Measure S:** The entangled beams are sent through polarizers, and coincidence counts are measured. By repeating this process for a set of 16 polarizer angles, S is measured.

5.1 Tuning the Bell State

The goal here is to use the QWP and HWP to tune the parameters θ and ϕ in the entangled state to as close to 45° and 0° as possible, so that something close to the Bell state is generated. As derived in [1], θ and ϕ can be expressed in terms of the number of (corrected) coincidence counts $N(a,b)$[1], where a and b are the angles of the polarizers with respect to the vertical:

[1]This corresponds to $N(a,b) - C$ in Dehlinger, where C is the statistical correction

$$\tan^2\theta = \frac{N(90^\circ,90^\circ)}{N(0^\circ,0^\circ)}, \tag{6}$$

$$\cos\phi = \frac{1}{\sin 2\theta}\left(4\frac{N(45^\circ,45^\circ)}{N(90^\circ,90^\circ)+N(0^\circ,0^\circ)} - 1\right). \tag{7}$$

Unfortunately, the polarizers used in this experiment are labelled so that the vertical and horizontal orientations don't correspond to 0° and 90°. The correct angles are tabulated here:

	Polarizer P1	Polarizer P2
$N(90^\circ,90^\circ)$	143°	50°
$N(0^\circ,0^\circ)$	53°	140°
$N(45^\circ,45^\circ)$	98°	95°

Note that the wave plates are very sensitive, so an adjustment by 1° will make a noticeable difference to the state. Also note that the HWP and QWP primarily control θ and ϕ respectively, but they are to some extent coupled so rotating one wave plate can affect both parameters. Each time one of the wave plates is adjusted, both parameters should be measured.

5.2 Measuring S

Once the state of the entangled light is close to the Bell state, the next step is to measure S. We repeat the formula here for convenience:

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b'). \tag{8}$$

In the ideal case in which the exact Bell state is used, S is maximized at $2\sqrt{2}$ for angles $a = -45^\circ$, $a' = 0^\circ$, $b = 22.5^\circ$, $b' = -22.5^\circ$, as pictured in Fig. 5.

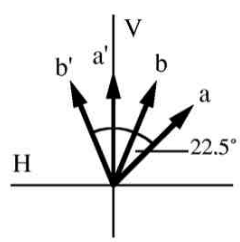

Figure 5: The four angles used to measure S. Taken from [1]

To measure $E(a,b)$, we must measure the four quantities $P_{VV}(a,b)$, $P_{HH}(a,b)$, $P_{VH}(a,b)$, and $P_{HV}(a,b)$. Recall that $P_{HH}(a,b)$ is the probability for both photons to be absorbed when the transmission axes of the polarizers are a and b with respect to the vertical (see FigXXX). This is not convenient to measure, because the single photon detectors measure how many photons are *transmitted*, not absorbed. However, we can use the fact that if we rotate both polarizers by 90°, then the probability for transmission is equal to $P_{HH}(a,b)$. Since the probability for both photons to be transmitted when the transmission axes are at a and b is $P_{VV}(a,b)$, we get the relation

$$P_{HH}(a,b) = P_{VV}(a_\perp, b_\perp), \tag{9}$$

where $a_\perp = a + 90^\circ$ and $b_\perp = b + 90^\circ$. This means that to measure the number of photons absorbed for given polarizer orientations, you just need to turn the polarizers by 90° and measure the number of photons transmitted. Similarly, we have $P_{HV}(a, b) = P_{VV}(a_\perp, b)$ and $P_{VH}(a, b) = P_{VV}(a, b_\perp)$.

We can now compute $E(a, b)$ from measurements of coincidence counts by using $P_{VV}(a, b) = N(a, b)/N_{tot}$, $P_{HH}(a, b) = N(a_\perp, b_\perp)/N_{tot}$, $P_{VH}(a, b) = N(a, b_\perp)/N_{tot}$, and $P_{HV}(a, b) = N(a_\perp, b)/N_{tot}$. Here $N_{tot} = N(a, b) + N(a_\perp, b_\perp) + N(a, b_\perp) + N(a_\perp, b)$. This gives us an expression for $E(a, b)$:

$$E(a, b) = \frac{N(a, b) + N(a_\perp, b_\perp) - N(a, b_\perp) - N(a_\perp, b)}{N(a, b) + N(a_\perp, b_\perp) + N(a, b_\perp) + N(a_\perp, b)} \tag{10}$$

Measuring S thus requires measuring coincidence counts for 16 sets of polarizer angles, 4 for each measurement of E.

6 Procedure

SAFETY REMINDERS

- Be aware that you are using a powerful class 3B laser in this lab. Direct exposure of the laser light to your eyes is hazardous.
- Do not open the lid to the black safety enclosure while the laser is on.
  Note: If the lid is opened during laser operation, a safety interlock will trip and turn the laser off for your safety. However, this measure is employed as a last resort, and should **not** be taken for granted.
- Do not attempt to disengage or in any way tamper with the interlock mechanism (the microswitch inside the box as well as the circuitry behind the laser controller). This may result in serious damage to the user as well as the system.
- If you have reason to believe that the interlock system is not working as it should, do not under any circumstances open the lid. Contact the APL staff immediately to have it checked and fixed.
- If you need to open the safety enclosure lid for any reason, make sure you are wearing safety goggles, even if you have turned the laser off. Again: do not take the interlock system for granted!
- Do not attempt to unscrew and/or remove the safety enclosure box from its location on the optical bench. It has been designed and positioned specifically to ensure that the entangled photons are not blocked, while ensuring your safety.
- Never put your eyes at the level of the laser.

NOTE: This is not a complete list of all possible hazards; we cannot warn against all possible creative stupidities. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, dont precariously balance heavy objects,.... If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. If an accident or incident happens, you must let us know. More safety information is available at http://www.ehs.utoronto.ca/resources.htm.

6.1 Aligning the laser

NOTE: a detailed walkthrough of the alignment procedure is given in flow-chart format in appendix A.

1. Make sure that the irises on the collectors are closed at all times when the experiment isn't being performed, particularly when the lights in the room are on. Make sure that you have your safety goggles on and that the safety enclosure is completely closed.

2. Open the software **detection.vi**. This is the main interface for the experimental measurements. See the description of LabVIEW components. Make the update window 1 s if it is not already this value.

3. Turn on the microcontroller, the power supply and the laser controller. Press the button directly below the knob to turn on the laser. Turn the current knob on the laser controller to obtain a current reading of 80 mA.

4. Before optimizing the alignment, it is necessary to make sure that the statistical corrected counts due to background light are being calculated correctly by the program. Turn the room's lights off and set the update period on the LabVIEW program to 1 s. Also, make sure the laser is still turned off. While shining the flashlight near (but not directly at) the collectors, adjust the coincidence window such that the system reports a corrected coincidence count that averages to near 0. If accomplished, this would mean that the software is doing its job in removing the effect of coincidences due to background light, since in this step there are no user-generated entangled photons in play. It may be useful to increase the update window to average over a longer time.

5. Check the single photon counts. If they are at least 5K for both detectors, skip the next step, and if they are above 30K, then skip the next two steps.

6. If the counts are less than 5K, use the alignment laser to align the beam. Turn off the main laser and all the lights in the room, then unscrew the laser fiber from one of the single photon detectors. Unscrew the front piece of the collector so that when you shine the alignment laser into the fiber (see Fig. 7), it can shine through the collector (there is a filter in the front part of the collector that will block light red light). Now open the safety enclosure (first double-checking the laser is off) and adjust the knobs of the collector so that the red dot is aligned to the center of the BBO crystal. Repeat this process for the other detector, then close the safety enclosure, carefully screw the front piece of the collectors back on, and reconnect the fiber to the single photon detectors.

7. You must now make careful adjustments (the setup is very sensitive) to the alignment of the mirrors and collectors. You should have one person read off the single photon count values while you slowly turn one knob. Stop when you think you have found the spot that maximizes the single photon counts, then move on to the next knob. Some knobs may already be in the optimal position and not require changing. Continue doing this until the single photon counts are at least 30K. If you are having trouble obtaining this number, speak to the TA.

8. Now check the coincidence counts. Note that high single photon counts does not necessarily mean the coincidence counts will be high. This is because single photon counts only measure how many photons are entering the detectors, whereas coincidence counts require the pairs of entangled photons from the two beams to enter the detectors simultaneously. Repeat the

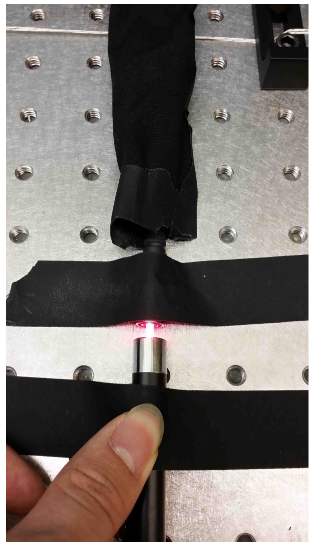

Figure 6: Shining the alignment laser in to the optical fiber.

process of step 3, but now trying to obtain coincidence counts of at least 100 (in 1 s window), while at the same time maintaining single photon counts of at least 30K. This may take some time as the setup is quite sensitive.

6.2 Tuning the state

1. First measure θ and ϕ without changing the orientation of the QWP or the HWP.
2. If θ and ϕ are relatively close to 45° and 0°, then move on to measuring S.
3. If this is not the case, then use trial and error with adjusting the HWP to make $N(90°, 90°)$ and $N(0°, 0°)$ approximately equal, yielding a θ value close to 45°.
4. Now use trial and error with adjusting the QWP to tune ϕ it as close to 0° as possible, while also monitoring θ to make sure it stays near 45°.

6.3 Measuring S

1. For this final step you must first measure $E(a, b)$ for angles $(-45°, 22.5°)$, $(-45°, -22.5°)$, $(45°, 22.5°)$, and $(45°, -22.5°)$. This is done by measuring coincidence counts and using Eq. 10.
2. S can then be calculated using Eq. 8.

7 Questions

1. Using equations found in [1], S can be computed theoretically for an entangled state with arbitrary θ and ϕ. Compute S for the values of θ and ϕ of the state you used. In theory, should this state violate Bell's inequality? How does this theoretical prediction correspond to what you measured for S?

2. What are potential sources of error in this experiment? Are you able to account for these errors in your final results? How could these errors be reduced? Based on theses errors, how confident are you of your final results?

References

[1] D. Dehlinger and M. W. Mitchell. "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory" *Am. J. Phys.* **70**, 903 (2002).

A Alignment flow-chart

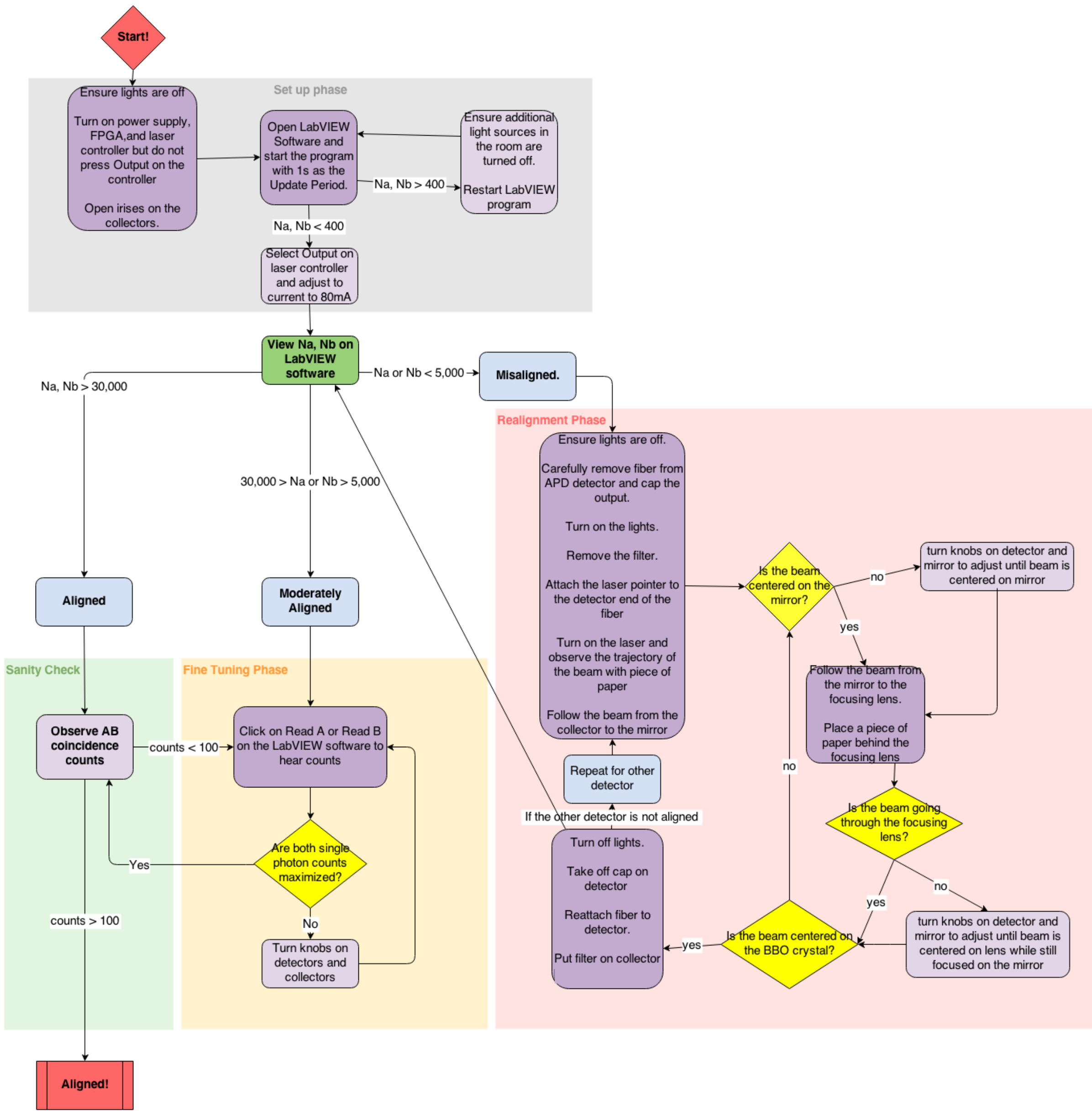

Figure 7: The flowchart used for alignment of the laser. Na and Nb refer to the single photon counts from channels A and B.